Solution for HW2 27-9-2016

§12) 2) $f(z) = z^3 + z + 1$

$$= (x+iy)^3 + (x+iy) + 1$$
$$= x^3 + 3ix^2y - 3xy^2 - iy^3 + x + iy + 1$$
$$= (x^3 - 3xy^2 + x + 1) + i(3x^2y - y^3 + y)$$

3) $f(z) = x^2 - y^2 - 2y + i(2x - 2xy)$

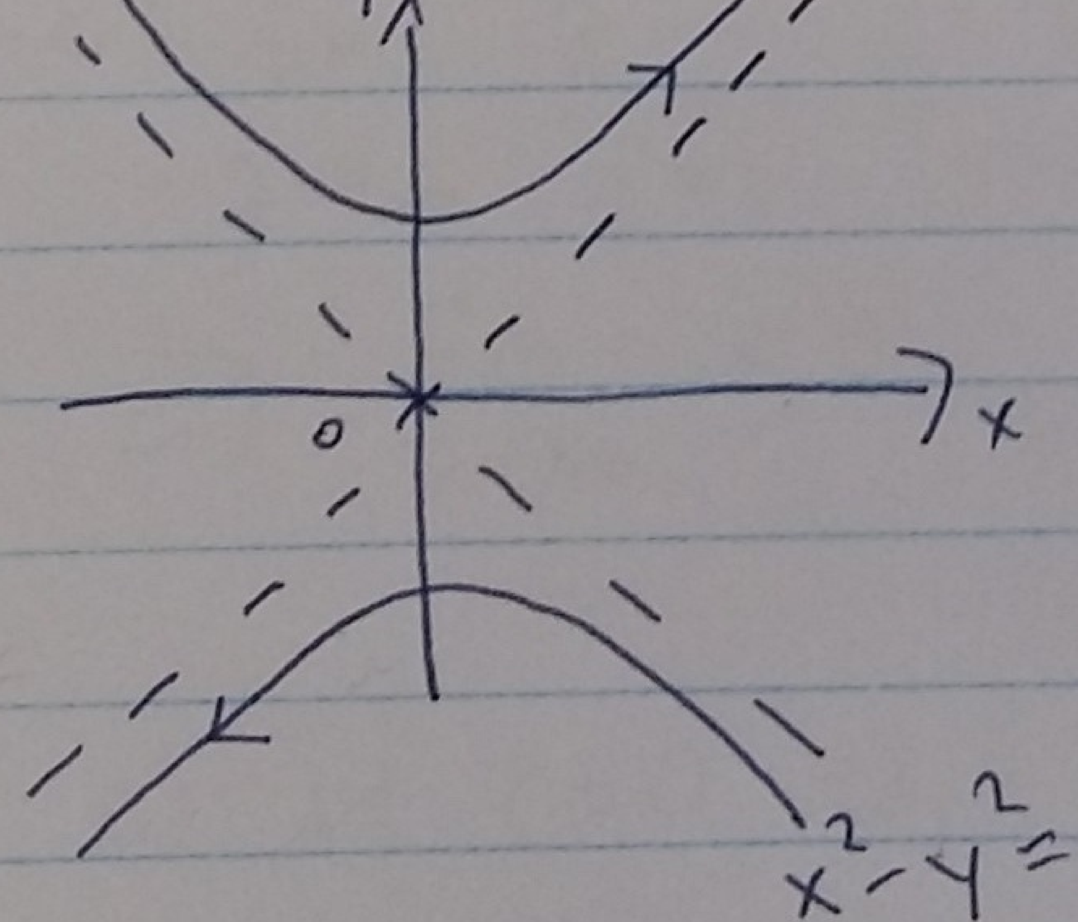
$$= \left(\frac{z+\bar{z}}{2}\right)^2 - \left(\frac{z-\bar{z}}{2i}\right)^2 - 2\left(\frac{z-\bar{z}}{2i}\right) + i\left(2\left(\frac{z+\bar{z}}{2}\right) - 2\left(\frac{z+\bar{z}}{2}\right)\left(\frac{z-\bar{z}}{2i}\right)\right)$$

$$= \frac{z^2 + 2|z|^2 + \bar{z}^2}{4} + \frac{z^2 - 2|z|^2 + \bar{z}^2}{4} - \frac{z-\bar{z}}{i} + i\left(z + \bar{z} - \frac{z^2 - \bar{z}^2}{2i}\right)$$

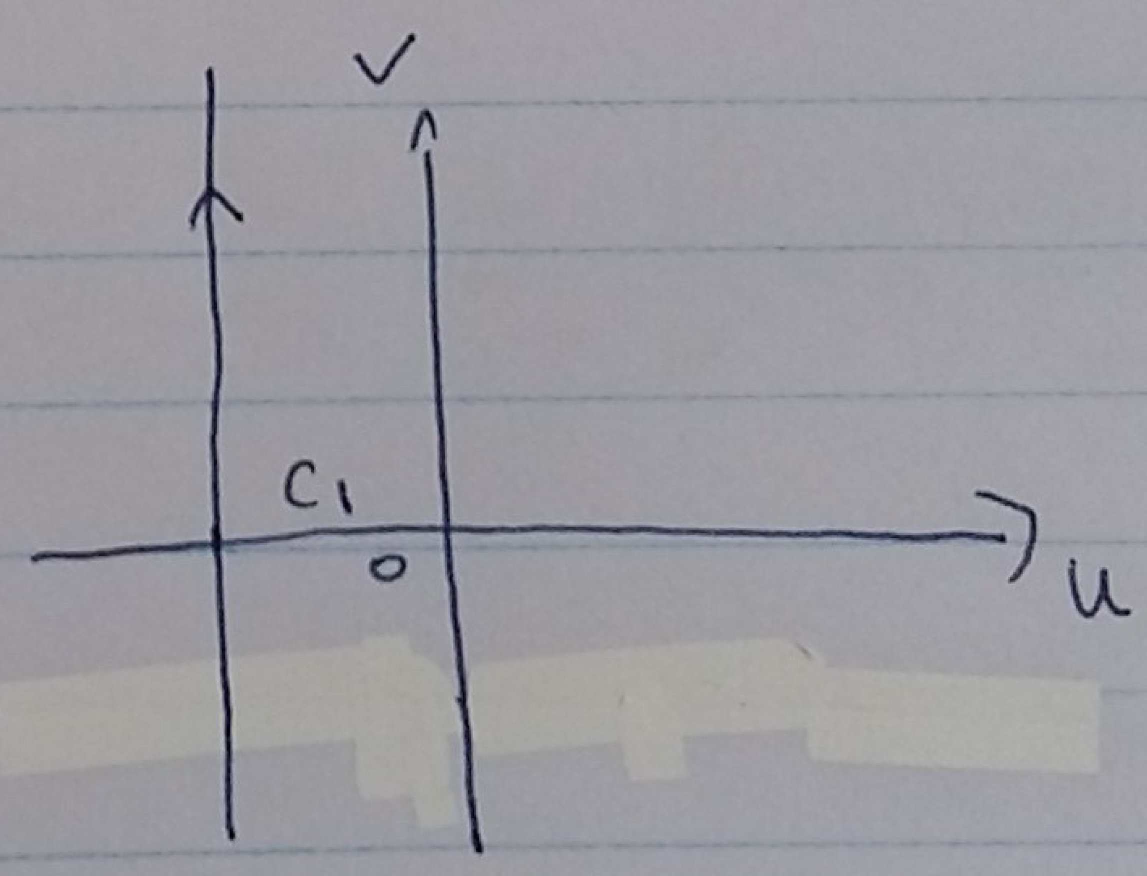
$$= \bar{z}^2 + 2iz$$

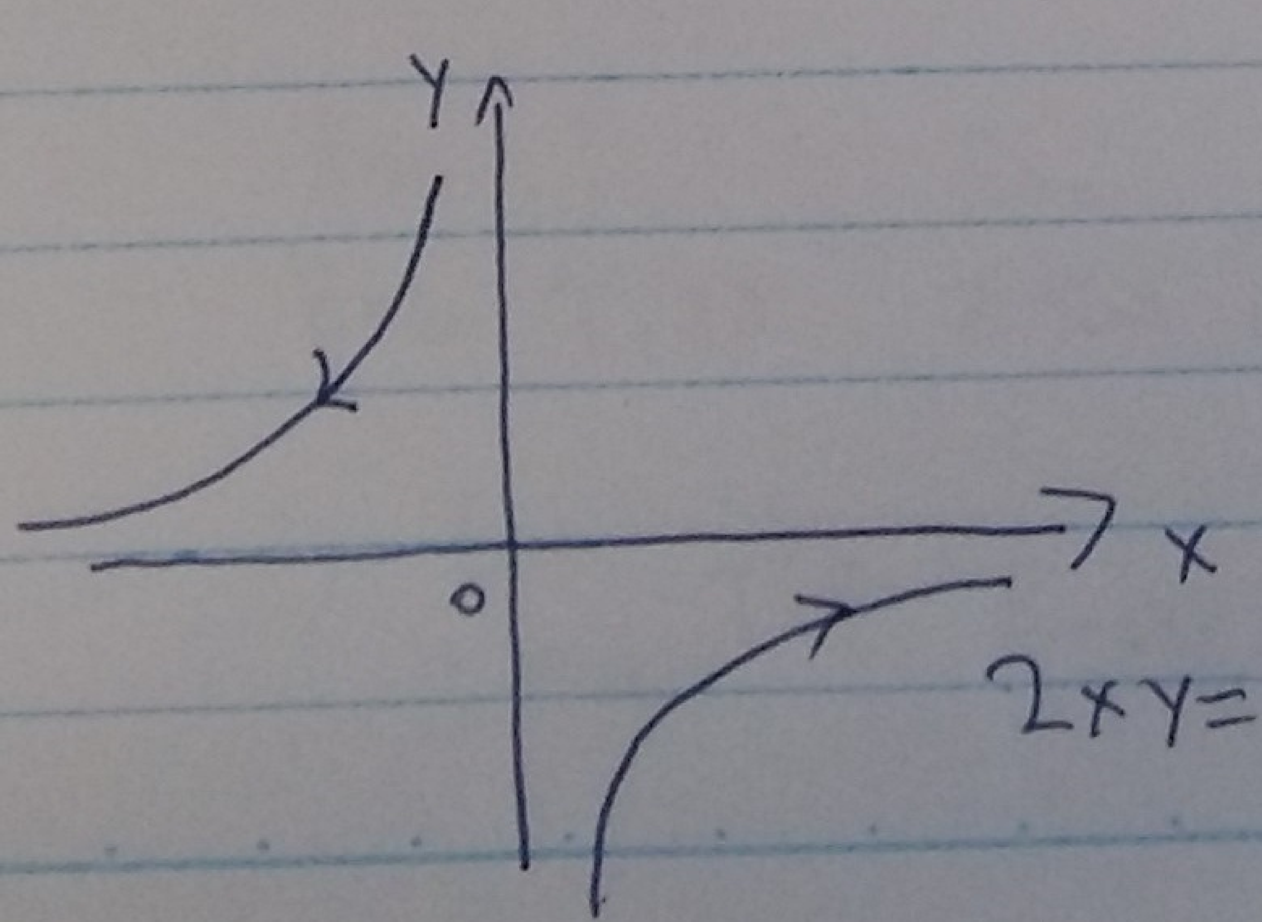
§14) 2) Note that $w = z^2 = (x+iy)^2 = (x^2 - y^2) + i(2xy)$.

So we have

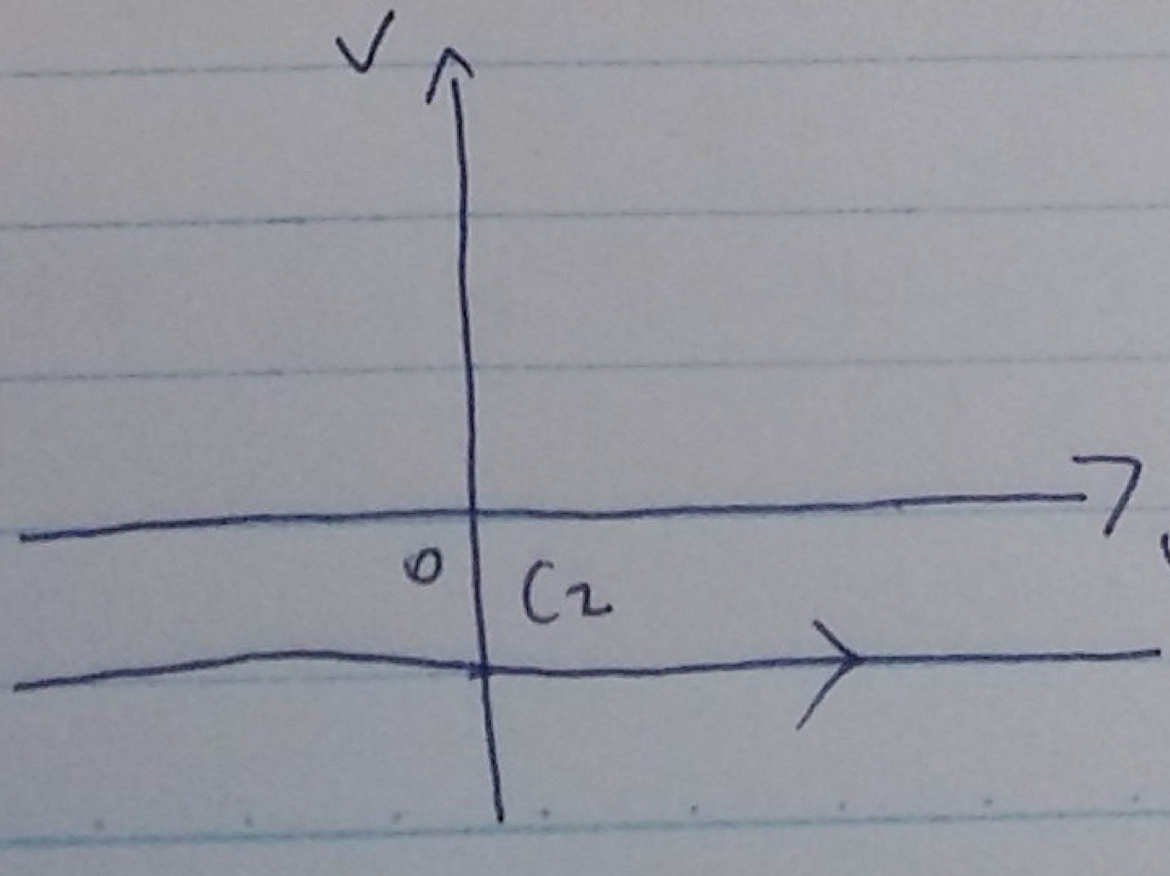


$w = z^2$

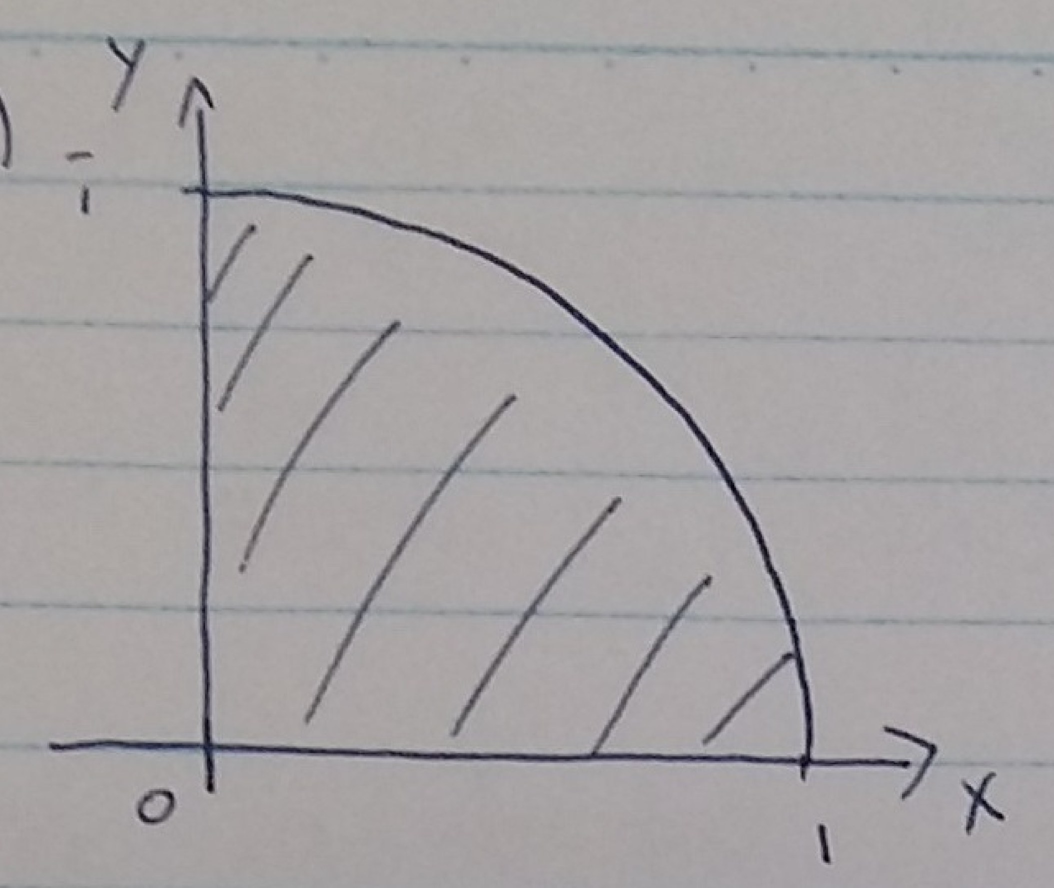




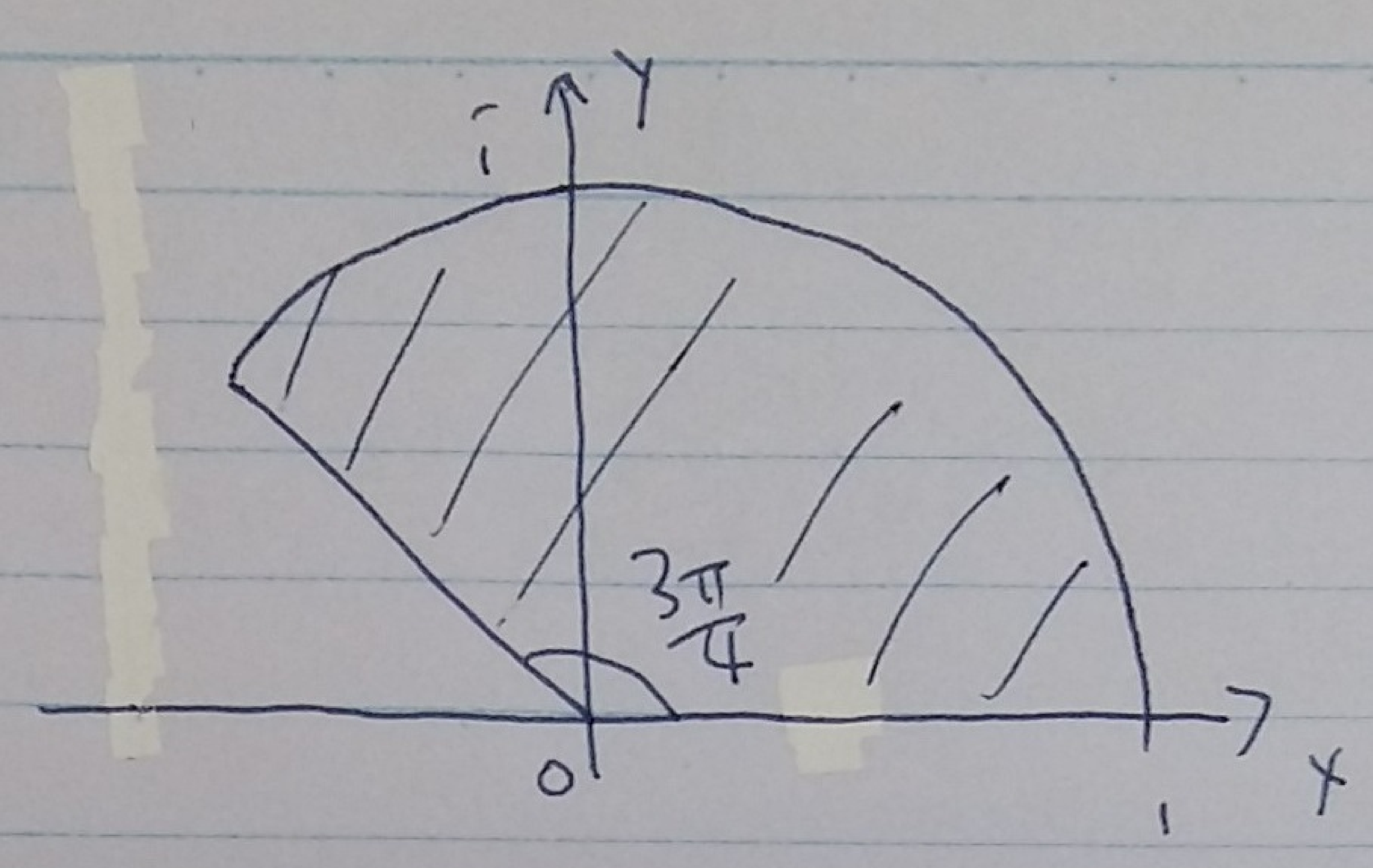
$w = z^2$



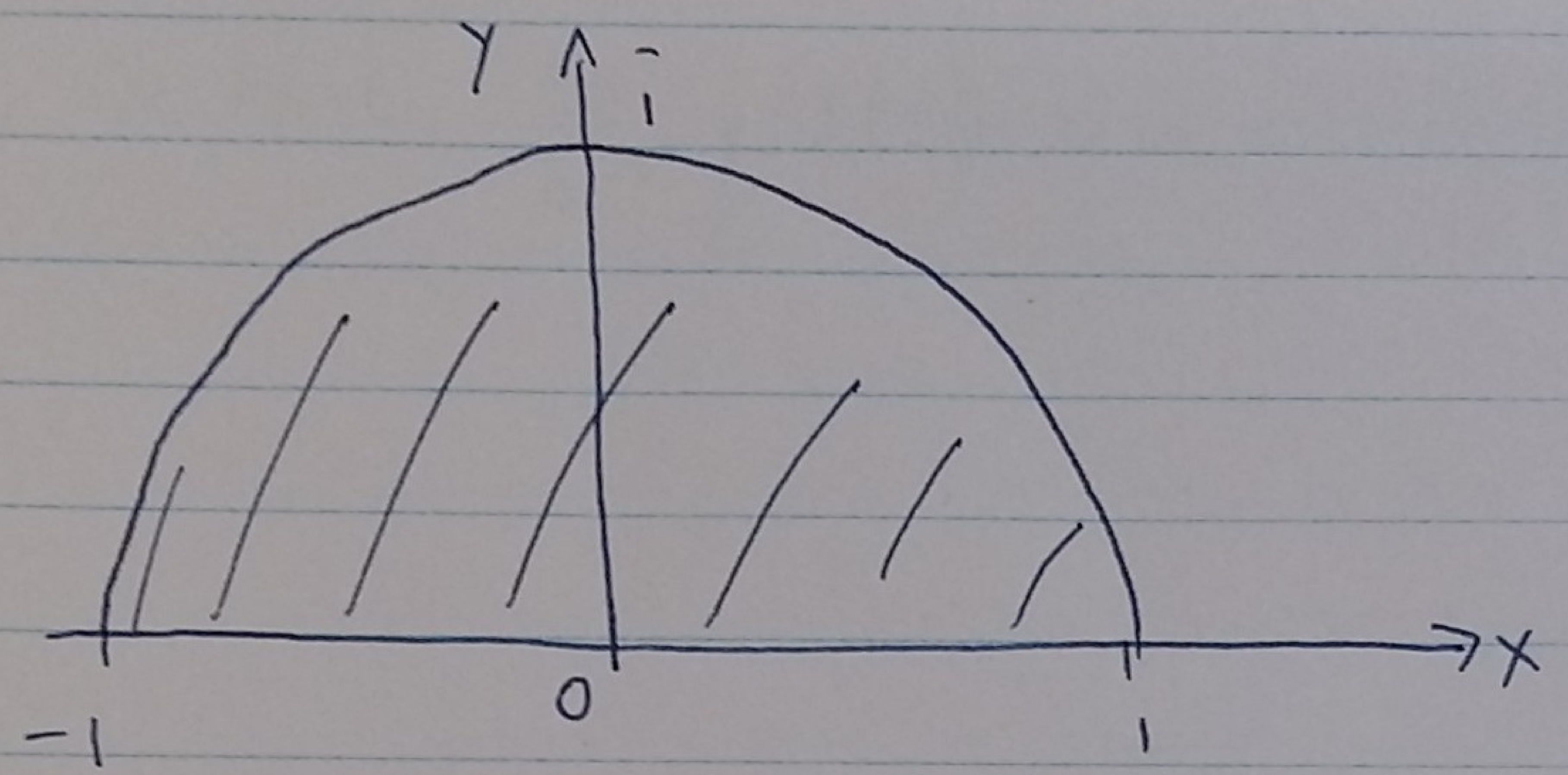
3) a)



b)



c)



§18) 10) a) $\displaystyle\lim_{z\to\infty} \frac{4z^2}{(z-1)^2} = \lim_{w\to 0} \frac{4\left(\frac{1}{w^2}\right)}{\left(\frac{1}{w}-1\right)^2} = \lim_{w\to 0} \frac{4}{(1-w)^2} = 4$

b) $\displaystyle\lim_{z\to 1} (z-1)^3 = 0 \Rightarrow \lim_{z\to 1} \frac{1}{(z-1)^3} = \infty$

c) $\displaystyle\lim_{z\to\infty} \frac{z-1}{z^2+1} = \lim_{w\to 0} \frac{\frac{1}{w}-1}{\frac{1}{w^2}+1} = \lim_{w\to 0} \frac{w-w^2}{1+w^2} = \frac{0}{1} = 0$

$\displaystyle\Rightarrow \lim_{z\to\infty} \frac{z^2+1}{z-1} = \infty$

11) a) If $c=0$, then $T(z) = \dfrac{az+b}{d}$. Note that $a \neq 0$.

$\displaystyle\lim_{z\to\infty} \frac{1}{T(z)} = \lim_{z\to\infty} \frac{d}{az+b} = \lim_{w\to 0} \frac{d}{\frac{a}{w}+b} = \lim_{w\to 0} \frac{dw}{a+bw} = 0$

$\displaystyle\Rightarrow \lim_{z\to\infty} T(z) = \infty.$

b) If $c \neq 0$, $\lim_{z \to \infty} T(z) = \lim_{w \to 0} \frac{\frac{a}{w}+b}{\frac{c}{w}+d} = \lim_{w \to 0} \frac{a+bw}{c+dw} = \frac{a}{c}$.

Also, $\lim_{z \to -\frac{d}{c}} \frac{1}{T(z)} = \lim_{z \to -\frac{d}{c}} \frac{cz+d}{az+b} = \frac{c(\frac{-d}{c})+d}{a(\frac{-d}{c})+b} = 0$

since $ad-bc \neq 0$.

Hence we have $\lim_{z \to -\frac{d}{c}} T(z) = \infty$.